Statistics

Summer 2023

Lecture 7

Feb 19-8:47 AM

Class QZ 7

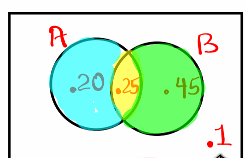
Given $\boxed{P(A) = .45}$, $\boxed{P(B) = .7}$ $\boxed{P(A \text{ and } B) = .25}$

1) $P(\bar{A}) = 1 - .45 = \boxed{.55}$ ✓
$= 1 - P(A)$

2) $P(A \text{ or } B) = .45 + .7 - .25 = \boxed{.9}$ ✓

Addition Rule
$= P(A) + P(B) - P(A \text{ and } B)$

3) Make Venn Diagram



Total = 1 ✓

$P(A \text{ only}) = .45 - .25 = \boxed{.2}$

$P(B \text{ only}) = .7 - .25 = \boxed{.45}$

$1 - (.20 + .25 + .45) = .1$

P(A or B, but not both) =
P(A only OR B only) = $.2 + .45 = \boxed{.65}$

$P(\bar{A} \text{ and } \bar{B}) = P(\overline{A \text{ or } B}) = 1 - P(A \text{ or } B) = 1 - .9 = \boxed{.1}$

De Morgan's Law

$P(\bar{A} \text{ or } \bar{B}) = P(\overline{A \text{ and } B}) = 1 - P(A \text{ and } B) = 1 - .25 = \boxed{.75}$

Jun 21-11:37 AM

Suppose $P(A) = .15$, $P(B) = .65$ and $A \,\&\, B$ are disjointed events.

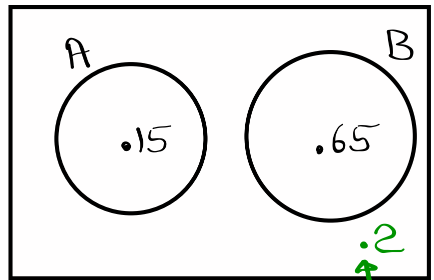
1) $P(\bar{A}) = 1 - P(A) = \boxed{.85}$

2) $P(\bar{B}) = 1 - P(B) = \boxed{.35}$

3) $P(A \text{ and } B) = 0$

NO overlap

4) Make Venn Diagram



√Total = 1

$1 - (.15 + .65) = .2$

5) $P(A \text{ or } B)$

$= P(A) + P(B) - P(A \text{ and } B) = .15 + .65 - 0 = \boxed{.8}$

Jun 22-7:42 AM

A box has 50 balls and 32 of them are red.

If we randomly select one ball,

1) $P(\text{Red}) = \frac{32}{50} = \boxed{.64}$

2) $P(\overline{\text{Red}}) = 1 - P(\text{Red}) = \boxed{.36}$

3) How many balls were not Red? 32

Total # balls → $50 - 32 = 18$

4) Find odds in favor of selecting a red ball.

$\#\text{Red} : \#\overline{\text{Red}}$

$32 : 18 \rightarrow \boxed{16:9}$

5) What about odds against selecting a red ball?

9 : 16

Jun 22-7:49 AM

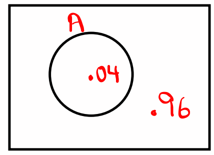
Suppose $P(A) = .04$

1) Is A a rare event? explain

Yes, $0 < P(A) \le .05$

2) $P(\bar{A}) = 1 - P(A) = \boxed{.96}$

3) Make Venn Diagram



Total = 1

4) write P(A) in reduced fraction.

.04 [Math] [1:▶Frac] [Enter] $\boxed{\frac{1}{25}}$

5) Find odds in favor of event A.

$P(A) : P(\bar{A})$

$.04 : .96 \Rightarrow \boxed{1 : 24}$

.04 [÷] .96 [Math] [1:▶Frac] [Enter]

$\frac{1}{24}$

6) Find odds against event A.

24 : 1

Jun 22-7:55 AM

Multiplication Rule

SG 12

Keyword: AND

multiple action events

$P(A \text{ and } B) = P(A) \cdot P(B|A)$

A happens first, then B happens

Given

1) Independent events

$\boxed{\text{If } A \,\&\, B \text{ are independent events, then } P(A \text{ and } B) = P(A) \cdot P(B)}$

What are independent events?

The outcome of one event, does not change the prob. of next event.

ex: $P(\text{Boy}) = .5$, $P(\text{Girl}) = .5$

It does not matter what the first born is, $P(\text{Next new born is boy}) = .5$

$P(\text{Next new born is girl}) = .5$

ex: $P(\text{Ace}) = \frac{1}{13}$ from a standard deck of cards.

If we do replacement,

$P(\text{Ace in any Selection}) = \frac{1}{13}$

ex: A multiple-choice exam, Each question has 5 choices, but only only one correct choice

making random guesses.

First Q. $P(\text{Correctly}) = \frac{1}{5}$ Third Q. $P(\text{Correctly}) = \frac{1}{5}$

Second Q. $P(\text{Correctly}) = \frac{1}{5}$ 4th Q. $P(C) = \frac{1}{5}$

Jun 22-8:10 AM

Given $P(A) = .2$, $P(B) = .5$

A & B are independent events

1) $P(\bar{A}) = \boxed{.8}$

2) $P(\bar{B}) = \boxed{.5}$

3) $P(A \text{ and } B) = P(A) \cdot P(B)$

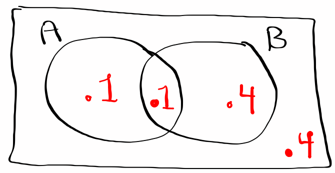
$= (.2) \cdot (.5) = \boxed{.1}$

4) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Addition Rule

$= .2 + .5 - .1 = \boxed{.6}$

5) Make Venn Diagram



✓Total = 1

Jun 22-8:22 AM

Suppose $P(A) = .3$, $P(B) = .6$

A & B are independent events

1) $P(\bar{A}) = 1 - P(A) = \boxed{.7}$

2) $P(\bar{B}) = 1 - P(B) = \boxed{.4}$

3) $P(A \text{ and } B) = P(A) \cdot P(B) = .3 \cdot .6 = \boxed{.18}$

4) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Addition Rule

$= .3 + .6 - .18 = \boxed{.72}$

5) $P(\bar{A} \text{ and } \bar{B}) = P(\overline{A \text{ or } B}) = 1 - .72 = \boxed{.28}$

De Morgan's law

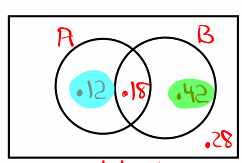
6) $P(\bar{A} \text{ or } \bar{B}) = P(\overline{A \text{ and } B}) = 1 - .18 = \boxed{.82}$

7) Make Venn Diagram

$P(A \text{ only}) = .3 - .18 = .12$

$P(B \text{ only}) = .6 - .18 = .42$

$1 - (.12 + .18 + .42) = .28$



Total = 1

8) $P(A \text{ only OR } B \text{ only}) = .12 + .42$

+

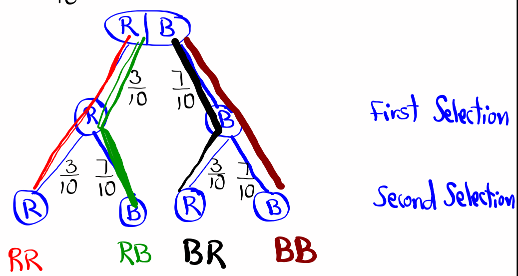
$= \boxed{.54}$

Jun 22-8:27 AM

A box has 3 Red and 7 Blue balls.

We Select 2 balls with replacement.

$P(\text{Red}) = \frac{3}{10}$, $P(\text{Blue}) = \frac{7}{10}$



$P(RR) = \frac{3}{10} \cdot \frac{3}{10} = \boxed{\frac{9}{100}}$

$P(RB) = \frac{3}{10} \cdot \frac{7}{10} = \boxed{\frac{21}{100}}$

$P(BR) = \frac{7}{10} \cdot \frac{3}{10} = \boxed{\frac{21}{100}}$

$P(BB) = \frac{7}{10} \cdot \frac{7}{10} = \boxed{\frac{49}{100}}$

$P(2 \text{ Reds}) = \frac{9}{100}$

$P(1R \,\&\, 1B) = \frac{42}{100}$

$P(\text{No Red}) = P(BB) = \frac{49}{100}$

# Red	P(# Red)
2	.09
1	.42
0	.49

Jun 22-8:41 AM

# Red	P(# Red)
2	.09
1	.42
0	.49

Red → L1

P(# Red) → L2

Use $\boxed{\text{1-Var Stats}}$ with

L1 & L2 to find

$\bar{x} = .6$

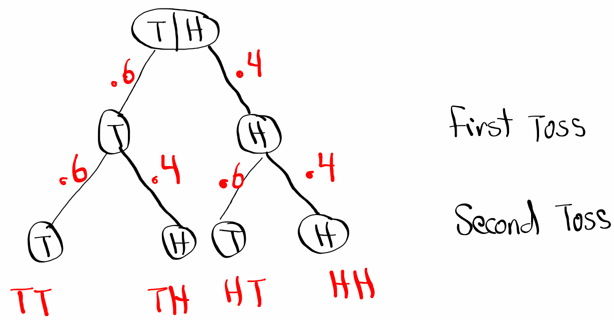
S = Blank ✓

$n = 1$ ← Total Prob.

Jun 22-9:12 AM

A loaded Coin is tossed 2 times

P(Tails) = .6 P(Heads) = .4



$P(TT) = (.6)(.6) = \boxed{.36}$

$P(TH) = (.6)(.4) = \boxed{.24}$

$P(HT) = (.4)(.6) = \boxed{.24}$

$P(HH) = (.4)(.4) = \boxed{.16}$

P(2 Tails) = .36

P(exactly 1 Tail) = .48

P(No tails) = .16

# tails	P(# Tails)
2	.36
1	.48
0	.16

Jun 22-9:16 AM

# tails	P(# Tails)
2	.36
1	.48
0	.16

Tails → L1

P(# Tails) → L2

Use $\boxed{\text{1-Var stats}}$ with

L1 & L2 To find

$\bar{x} = 1.2$

S = Blank

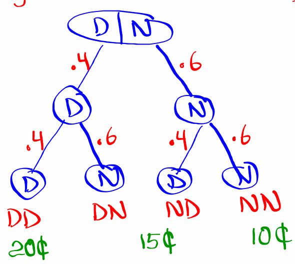
$n = 1$

Jun 22-9:23 AM

A box has 2 dimes & 3 nickels.

we take $\boxed{2 \text{ Coins}}$ with replacement

$P(\text{Dime}) = \frac{2}{5} = .4$ $\quad$ $P(\text{Nickel}) = \frac{3}{5} = .6$



$P(20¢) = P(DD) = (.4)(.4) = \boxed{.16}$

$P(15¢) = P(DN \text{ or } ND) = 2(.6)(.4) = \boxed{.48}$

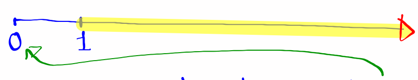
$P(10¢) = P(NN) = (.6)(.6) = \boxed{.36}$

Total ¢	P(Total ¢)
20	.16
15	.48
10	.36

Total(¢) → L1
P(Total(¢)) → L2
use $\boxed{\text{1-Var Stats}}$ with
L1 & L2 to find
$\bar{x} = 14$
S = Blank
n = 1

Jun 22-9:28 AM

Prob. with at least 1:
≥ 1



P(at least 1) = 1 − P(None)
Total
Prob.

A True-False QZ with $\boxed{3 \text{ questions}}$

making random guesses.

$P(\text{Correct}) = \frac{1}{2} = .5$ $\quad$ $P(\text{Wrong}) = \frac{1}{2} = .5$

W W W
Some W
&
Some C
C C C

P(All wrong)=
(.5)(.5)(.5) = .125

P(All Correct)=
(.5)(.5)(.5) = .125

P(at least 1 Correct ans)=

$1 - P(\text{All wrong}) = 1 - .125$

$= \boxed{.875}$

Jun 22-9:40 AM

A box has 2 Red & 8 Blue balls.

Select $\boxed{3 \text{ balls}}$ with $\boxed{\text{replacement}}$

on any Selection

$P(\text{Red}) = \frac{2}{10} = \boxed{.2}$ $P(\text{Blue}) = \frac{8}{10} = \boxed{.8}$

R R R

Some R & Some B

B B B

$P(RRR) = (.2)(.2)(.2) = \boxed{.008}$

$P(BBB) = (.8)(.8)(.8) = \boxed{.512}$

$P(\text{at least 1 Red}) = 1 - P(\text{No Red})$
$= 1 - P(BBB)$
$= 1 - .512$
$= \boxed{.488}$

$P(\text{at least 1 Blue}) = 1 - P(\text{No B})$
$= 1 - P(RRR) = 1 - .008 = \boxed{.992}$

Jun 22-9:51 AM

Dependent Events

one outcome changes prob. of next outcome.

If A and B are dependent events,

then $P(A \text{ and } B) = P(A) \cdot P(B|A)$

Given

A box has 4 Red & 6 Blue balls.

Select 2 balls, NO replacement

$P(RR) = \frac{4}{10} \cdot \frac{3}{9} = \boxed{\frac{12}{90}}$

$P(1R\ 1B) = P(RB \text{ or } BR) = 2\left(\frac{4}{10} \cdot \frac{6}{9}\right) = \boxed{\frac{48}{90}}$

$P(\text{No R}) = P(BB) = \frac{6}{10} \cdot \frac{5}{9} = \boxed{\frac{30}{90}}$

# Red	P(# Red)
2	12/90
1	48/90
0	30/90

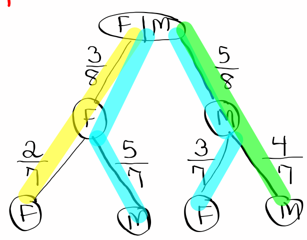
Red → L1
P(# Red) → L2
Use $\boxed{\text{1-Var Stats}}$
with L1 & L2

$\bar{x} = .8$
S = Blank
$n = 1$

Jun 22-10:25 AM

There are 3 Females & 5 males.
we need to Select 2 people.

Sample Space FF FM MF MM



First Selection

P(2 Females) $= \frac{3}{8} \cdot \frac{2}{7} = \boxed{\frac{6}{56}}$

P(exactly 1 Female) $= 2\left(\frac{3}{8} \cdot \frac{5}{7}\right) = \boxed{\frac{30}{56}}$

P(No Females) $= \frac{5}{8} \cdot \frac{4}{7} = \boxed{\frac{20}{56}}$

# F	P(# F)
2	6/56
1	30/56
0	20/56

$\#F \to L1$ $P(\#F) \to L2$

Use $\boxed{\text{1-Var Stats}}$ with L1 & L2 to find

$\bar{x} = .75$
$S =$ Blank
$n = 1$

Jun 22-10:36 AM

Consider a standard deck of playing cards.
Draw 3 Cards without replacement.

P(3 Aces) $= \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} = \frac{1}{5525}$

4 $\boxed{\div}$ 52 $\boxed{\times}$ 3 $\boxed{\div}$ 51 $\boxed{\times}$ 2 $\boxed{\div}$ 50
$\boxed{\text{Math}}$ $\boxed{\text{1:▸Frac}}$ $\boxed{\text{Enter}}$

P(No Aces) $= \frac{48}{52} \cdot \frac{47}{51} \cdot \frac{46}{50} = \boxed{\frac{4324}{5525}}$ ✓

P(at least 1 Ace) = 1 − P(No Aces)

Total Prob. = 1

$= 1 - \frac{4324}{5525} = \boxed{\frac{1201}{5525}}$

AAA
Some A & Some $\bar{A}$
$\bar{A}\bar{A}\bar{A}$

Jun 22-10:48 AM

Counting Method:

Select a number from 0 To 9.

0, 1, 2, 3, - - -, 9

10 choices.

Select 2 numbers with repetition from 0 To 9.

00	01	02	- - - -	09
10	11	12	- - - -	19
⋮				
90	91	92	- - - -	99

10 choices for first selection

10 " " Second "

$10 \cdot 10 = 100$ Total Selections

Your ATM card passcode

$\underline{10}\ \underline{10}\ \underline{10}\ \underline{10} = 10000$ Choices with repetition

$\underline{10}\ \underline{9}\ \underline{8}\ \underline{7}$ NO repetition

5040

Jun 22-11:15 AM

5 people, Select 2 people

Adam, Bill, Carol, David, Eddie

~~AB~~	~~AC~~	~~AD~~	~~AE~~
BA	~~BC~~	~~BD~~	~~BE~~
CA	CB	~~CD~~	~~CE~~
DA	DB	DC	~~DE~~
EA	EB	EC	ED

20 Total Selections

Suppose order does not matter

10 Total Selections

n different items, Select r of them, NO replacement, order does not matter

$$\#\text{ of Selection } \quad {}_nC_r = \frac{n!}{r! \cdot (n-r)!}$$

n choose r

Combination

$${}_5C_2 = \frac{5!}{2!\,(5-2)!} = \frac{5!}{2! \cdot 3!} = \boxed{10}$$

5 $\boxed{\text{MATH}}$ $\boxed{\text{PRB}}$ $\boxed{3{:}\,{}_nC_r}$ 2 $\boxed{\text{Enter}}$

Jun 22-11:21 AM

A box has 8 different balls, we need 3 of them. No replacement
Order does not matter.

how many ways can this be done?

$_8C_3 = \boxed{56}$

A basketball Coach has 12 players, and he needs 5 to Start the game.

how many ways Can this be done?

$_{12}C_5 = 792$

CA Lotto

Select 5 numbers in any order, no replacement, from 1 to 50, then Select a letter from A to Z.

How many ways can this be done?

$\underbrace{_{50}C_5}_{\text{numbers}} \cdot \underbrace{_{26}C_1}_{\text{letter}} = \boxed{55{,}087{,}760}$

Jun 22-11:29 AM

Class QZ 8

Given $P(A) = .4$, $P(B) = .8$, $P(A \text{ and } B) = .3$

find

1) $P(A \text{ or } B) = .4 + .8 - .3 = \boxed{.9}$

2) $P(\bar{A} \text{ and } \bar{B})$

$= P(\overline{A \text{ or } B}) = 1 - .9 = \boxed{.1}$

3) $P(\bar{A} \text{ or } \bar{B})$

$= P(\overline{A \text{ and } B})$

$= 1 - .3 = \boxed{.7}$

Jun 22-11:42 AM